RAM Math Circle - Chennai Session 5

Session format:

- 60 minutes: Counting problems
- 30 minutes: Geometry with paper folding

1 Introduction to counting

We continued working on counting problems based on the addition and multiplication principles.

Addition principle: If there are m different elements in set A and n different elements in set B, then the total number of ways of choosing 1 element from either set A or set B is m + n.

Multiplication principle: If there are m different elements in set A and n different elements in set B, then the total number of ways of choosing an element from set A and an element from set B is $m \times n$.

- 1. A store carries 8 styles of pants. For each style there are 10 different possible waist sizes, 6 pant lengths and 4 colour choices. How many different types of pants could the store have?
- 2. In how many ways may one right and one left shoe be selected from 6 pairs of shoes without obtaining a pair?
- 3. There are 4 different roads from Alwarpet to T.Nagar, 3 different roads from T.Nagar to Marina, and 2 different roads from Alwarpet to Marina directly.
 - (a) How many different routes from Alwarpet to Marina altogether?
 - (b) How many different routes from Alwarpet to Marina and back if any road can be used once in each direction (i.e. the route can be repeated on the return journey)?
 - (c) How many different routes from Alwarpet to Marina and back that do not use any road twice?
 - (d) How many different routes from Alwarpet to Marina and back that visit T.Nagar at least once?
- 4. How many four-letter 'words' (sequences of letters with repetition, that may or may not make sense in English) are there in which the first and last letters are vowels?
- 5. How may ways are there to roll two dice to yield a sum divisible by 3?

2 Geometry with paper folding

We started constructing an octahedron using modular origami. We learned a basic construction called 'waterbomb' and learned to put 3 waterbomb modules together using the pocket-and-flap method. We will complete the construction next week.

3 Food for thought

- 1. How many times is the digit 0 written when listing all numbers from 1 to 3333?
- 2. A binary sequence of length n is a sequence in which each term is either 0 or 1. For example, here are all binary sequences of length 3:

000	0	1	1
$0 \ 0 \ 1$	1	1	0
$0\ 1\ 0$	1	0	1
$1 \ 0 \ 0$	1	1	1

- (a) How many binary sequences of length 4 are there? (You may or may not list them all.)
- (b) How many binary sequences of length 5?
- (c) How many binary sequences of length n? (Here n can be any positive integer, so your answer should be a formula in terms of n).

