September 22 2024

RAM Math Circle - Chennai

Session 4

Session format:

- 60 minutes: Counting problems
- 30 minutes: Geometry with paper folding

1 Introduction to counting

We worked on a number of simple problems to introduce counting strategies. At the end of the session, we noted the addition and multiplication principles for counting (in the simplest case).

1. Given 3 different English books, 5 different Science books and 4 different Math books, in how many ways can you choose 1 book?
2. In how many ways can we choose a number from $1, 2, 3, 4, 5, 6$ which is either a perfect square or a multiple of 3?
3. In a classroom of 12 boys and 15 girls, in how many ways can a teacher choose 1 student for show-and-tell?
4. A restaurant offers 5 starter, 3 salads and 7 mains.
   (a) In how many ways can you choose 1 item (either a starter or a salad or a main dish)?
   (b) How many different meals with 1 starter and 1 main are possible?
   (c) How many different meals with 1 starter, 1 salad and 1 main are possible?
5. There are 5 different brands of waveboards and 4 different brands of bicycles at your local sports store.
   (a) If you can buy only 1 thing, how many choices do you have?
   (b) In how many ways can you buy 1 bicycle and 1 waveboard?

Addition principle: *If there are m different elements in set A and n different elements in set B, then the total number of ways of choosing 1 element from either set A or set B is $m + n$.*

Multiplication principle: *If there are m different elements in set A and n different elements in set B, then the total number of ways of choosing an element from set A and an element from set B is $m \times n$.*

2 Geometry with paper folding

We looked at 3 simple constructions and how to reason the working of these constructions using Euclidean geometry.

1. Making a square from a rectangle
2. Folding an isosceles triangle from a square
3. Folding an equilateral triangle from a square

Finally we looked at a geometric proof of the fact that the series $\sum_{i=1}^{\infty} \frac{1}{2^i}$ converges to 1.

3 Food for thought

1. Any crease through the center of the square divides it into two trapezoids which are congruent. A second crease through the center, which is at right angles with the first, divides the square into four congruent quadrilaterals, of which two opposite angles are right angles. All these quadrilaterals are cyclic. (Adapted from T.Sundara Row's book)

2. Given 5 different English books, 3 different French books and 4 different German books:

   (a) How many different ways are there to select 3 books, one of each language?

   (b) How many ways are there to make a row of 3 books in which exactly 1 language is missing, if the order of the books makes a difference?